

# A model of mesons in finite extra-dimension

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## Abstract

Recently, problem of stability of H-atom has been reported in extra-finite dimension, and found out that it is stable in extra-finite dimension of size,  $R \leq \frac{a_0}{4}$ , where,  $a_0$  is the Bohr radius. Assuming that, the heavy flavoured mesons have also such stability controlled by the scale of coupling constant, we obtain corresponding QCD Bohr radius and it is found to be well within the present theoretical and experimental limit of higher dimension. We then study its consequences in their masses using effective string inspired potential model in higher dimension pursued by us. Within the uncertainty of masses of known Heavy Flavoured mesons the allowed range of extra dimension is  $L \leq 10^{-16}m$ , which is well below the present theoretical and experimental limit, and far above the Planck length  $\simeq 1.5 \times 10^{-35}$  m.

Keywords: Luscher term, compact extra dimension, QCD.

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## 1 Introduction

There has been considerable discussion on the possibility of extra spatial dimension since long ago. The idea was born in 1920's when Kaluza and Klein[1] for the first time introduced one additional dimension. Later different theories such as ADD[2], RS[3], UED[4] etc were introduced supporting the idea. In recent years, equally interesting research is going on what happens to the ordinary H-atom, when there is more than 3 dimension. The problem of stability of H-atom in a compact space was a subject of research in[6] by Bures and Seigl. They reported that H-atom is

stable in extra-finite dimension of size,  $R \leq \frac{a_0}{4}$ , where,  $a_0$  is the Bohr radius. Assuming that, the heavy flavoured mesons have also such stability controlled by the scale of coupling constant, we obtain corresponding QCD Bohr radius,  $R_{QCD} \leq \frac{a_0|_{QCD}}{4}$ , where  $a_0|_{QCD} = \frac{3}{16\mu\alpha_s}$ ,  $\alpha_s$  is the strong coupling constant and  $\mu$  is the reduced mass of mesons.

Recently, a string inspired potential model[5] of mesons has been reported in higher dimension of infinite extent, assumed by us. In this work we assume the stability of mesons within their QCD Bohr radii in extra-dimension, and we study the masses of heavy flavoured mesons within this model. The additional assumption is that the modification of Coulomb potential in finite extra-dimension in a plausible way. Plausible relationship between confinement effect and modified coulomb potential in finite extra-dimension is also suggested.

In section.2 we outline the formalism, in section.3 we calculate the masses of Heavy Flavoured mesons in this model without confinement, while section.4 is devoted to numerical analysis. The last section.5 includes conclusion and discussion.

## 2 Formalism:

### 2.1 The string inspired potential model and its limitations:

Recently a String inspired potential model[5] is reported which assumes Luscher term[9],[5] to be the higher dimensional correction to Cornell potential[5],[7],

$$v(r) = \frac{-\gamma}{r} + br + c \quad (1)$$

where b is standard confinement parameter and

$$\gamma = \frac{\pi(d-2)}{24} \quad (2)$$

At d=3,

$$\gamma = \frac{4\alpha_s}{3} \implies \alpha_s = 0.0981 \quad (3)$$

This coupling constant does not correspond to its numerical value for d=3. Following MS-bar scheme using standard equation of coupling constant,

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2n_f}{3})(\ln \frac{Q^2}{\Lambda_{(QCD)}^2})} \quad (4)$$

The value given in equation(3) is much smaller than the corresponding value  $\alpha_s = 0.39$  at c-scale and  $\alpha_s = 0.22$  at b-scale. For  $n_f = 6, \Lambda_{QCD} = 0.382 GeV$ , the value of  $\alpha(Q)^2$  of equation(3) is reached at  $Q^2 = 1.2 \times 10^{13} GeV^2$ . Further for  $n_f = 16, \Lambda_{QCD} = 0.382 GeV$  we get theoretically allowed highest value of flavour  $Q^2 \simeq 5.7 \times 10^{(20)} GeV^2$ . So higher dimensional extension of Coulomb term does not correspond to standard QCD in present energy regime and hence ruled out.

## 2.2 Improved potential model in finite extra-dimension:

We consider the known 3 dimensions to be in the range 0 to  $\alpha$  and the extra dimension to be finite within the range 0 to  $L[3,4,6]$ . Thus,

$$r_d^2 = r_1^2 + r_2^2 + r_3^2 + y^2 \quad (5)$$

$$= r^2 + y^2 \quad (6)$$

where  $r^2 = r_1^2 + r_2^2 + r_3^2$ ,  $y$  is the size of finite extra dimension. Hence,

$$r_d \simeq r + \frac{y^2}{2r} \quad (7)$$

Now, we consider the Coulomb potential in d-dimension,

$$V(r_d) = -\frac{A}{r_d} \quad (8)$$

where,  $A = \frac{4\alpha_s}{3}$ , at d=3 And, with finite extra-dimension we modified it to,

$$\frac{4\alpha_s}{3} \longrightarrow \frac{4\alpha_s}{3} e^{-\mu_L y} \quad (9)$$

$$= \frac{4\alpha_s}{3}(1 - \mu_L y) \quad (10)$$

for  $\mu_L y \ll 1$ . At  $d = 3, y = 0$ , and we get back the standard 3-dimensional QCD coupling constant. The mass parameter ( $\mu_L$ ) occurred in equation(10) will be determined from the correspondence with the H-atom [13].

### 2.3 Wave function(D-Dimensional, with only Coulomb term in finite extra dimension):

The D-dimensional Schrodinger equation is [10], [14],

$$\left[ \frac{d^2}{dr^2} + \frac{d-1}{r_d} \frac{d}{dr} - \frac{l(l+d-2)}{r_d^2} + \frac{2\mu_L}{\hbar^2} (E - V_0) \right] R(r_d) = 0 \quad (11)$$

for  $l=0$ , taking  $\hbar = 1$ , we get

$$\ddot{R}(r_d) + \frac{d-1}{r_d} \dot{R}(r_d) + 2\mu_L \left( E + \frac{A}{r_d} \right) R(r_d) = 0 \quad (12)$$

Let,  $R(r_d) = F(r_d)e^{-\mu_L A r_d}$  [10], [15], Now putting  $R(r_d)$  in equation (12) we get,

$$\ddot{F}(r_d) + \left( \frac{d-1}{r_d} - 2\mu_L A \right) \dot{F}(r_d) + \left( \mu_L^2 A^2 - \frac{d-1}{r_d} \mu_L A + 2\mu_L E + \frac{2\mu_L A}{r_d} \right) F(r_d) = 0 \quad (13)$$

Now, we consider the series expansion of  $F(r_d)$  as,  $F(r_d) = \sum_{n=0}^{\alpha} a_n r_d^n f(r_d, d)$ , such that  $f(r_d) = 1$  at  $d = 3$ . Let us consider,  $f(r_d) = r^{\frac{\sigma(d-3)}{2}}$  [15], which satisfies this condition. Then the radial wave function can be expressed as,  $R(r_d) = \sum_{n=0}^{\alpha} a_n r_d^{n+\frac{\sigma(d-3)}{2}} e^{-\mu_L A r_d}$ . For ground state,  $n = 0$ , we get the unperturbed wave function,

$$\psi(r_d) = N_d (r^2 + y^2)^{\frac{\sigma(d-3)}{2}} e^{-\mu_L A (r + \frac{y^2}{2r})} \quad (14)$$

Now, at  $d = 3, y = 0$  and we get from above equation(14),

$$\psi(r) = N e^{-\mu_L A r} \quad (15)$$

which is consistent with standard H-atom wave function[13] at  $d=3$ , with  $A = \frac{1}{\mu\alpha}$ . Also, for the consistency of the wave function with H-atom wave function  $\mu_L$  corresponds to reduced mass  $\mu$  of H-atom.

## 2.4 Normalization: with 3 non-compact and one compact extra dimension

The normalization condition[16] is,

$$\int_0^\alpha \int_0^L dC_d (r^2 + y^2)^{\frac{\sigma(d-3)}{2}} |\psi(r, y)|^2 dr dy = 1 \quad (16)$$

where,  $C_d = \frac{(\pi)^{\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)}$ . The normalization constant obtained from the above equation has got singularity at  $d = 3$ , but for  $d = 3, L = 0$ , and it is free of singularity. At  $d = 3$ , in analogy with H-atom[13],  $N_H = (\frac{\mu^3 \alpha^3}{\pi})^{\frac{1}{2}}$ , standard QCD normalization constant is,

$$N_s = \left[ \frac{(\mu^{\frac{4}{3}} \alpha_s)^3}{\pi} \right]^{\frac{1}{2}} \quad (17)$$

Now, putting equation (14) in equation (16) we get (neglecting higher orders of  $L$ ),

$$N_d = \left[ \frac{(2\mu A)^{(2\sigma + \frac{3}{2})}}{d C_d (\frac{\pi}{\mu A})^{0.5} \Gamma(2\sigma + \frac{3}{2})} \right]^{\frac{1}{2}} \quad (18)$$

At  $d=3, A = \frac{4\alpha_s}{3}$ , putting (17) in (18) we get,

$$\left[ \frac{(\mu^{\frac{4}{3}} \alpha_s)^3}{\pi} \right]^{\frac{1}{2}} = \left[ \frac{2^{2\sigma+1.5}}{4\pi^2 \Gamma(2\sigma + \frac{3}{2})} \right] \left[ \left( \frac{4}{3} \alpha_s \mu \right)^{2\sigma+2} \right]^{\frac{1}{2}} \quad (19)$$

. To determine  $\sigma$  we need specific values of  $\mu = \frac{m_Q m_{\overline{Q}}}{m_Q + m_{\overline{Q}}}$  and  $\alpha_s$ . Unlike  $H_2$  atom reduced masses of mesons are flavour dependent and coupling constant is  $Q^2$  dependent. Hence, the correct wave function in compact extra dimension with only Coulomb term in the potential,

$$\psi(r_d) = N_d(r_d)^{\frac{\sigma_f(d-3)}{2}} e^{-\mu A(r + \frac{y^2}{2r})} \quad (20)$$

This is an improvement over the earlier result[5],[7],[10],where  $\sigma$  is assumed to be 1.To determine  $\sigma$  we need specific numerical values of  $\mu$  and  $\alpha_s$ .Unlike  $H$  atom reduced mass of mesons are flavour dependent and coupling constant is  $Q^2$  dependent.The values of  $\sigma$  for some Heavy flavoured mesons are shown in table(3).

Table 1: Variation of  $\sigma$  with reduced mass  $\mu$

$\mu(Gev)$	Meson	$\sigma$
0.2099	$D(c\bar{s})$	0.071
0.23727	$B(s\bar{b})$	0.083
1.02254	$B(b\bar{c})$	0.3489
2.33	$\tau(b\bar{b})$	0.796

### 3 Masses of heavy flavoured mesons in compact extra-dimension without confinement:

#### 3.0.1 Mass only with coulomb term in compact extra dimension:

As a application of the formalism developed in 2, we calculate the masses of Heavy flavoured mesons. Pseudoscalar meson mass can be computed from the following relation[12],[17]:

$$M_p = m_Q + m_{\bar{Q}} + \Delta E \quad (21)$$

where, $\Delta E = \langle H \rangle$ . In D-spatial dimension,the Hamiltonian operator H has the form[5],[18]:

$$H = -\frac{\nabla_d^2}{2\mu} + V(r_d) \quad (22)$$

where, $\mu =$  is the reduced mass of the meson with  $m_Q$  and  $m_{\bar{Q}}$  are the quark and anti-quark masses; $V(r_d)$  corresponds to the coulomb plus confinement term with plausible modification to compact extra dimension given as

$$V(r_d) = -\frac{A}{r_d} + br_d \quad (23)$$

In this work we first neglect the confinement term and see the effect of coulomb term with modification in compact extra dimension, and  $\nabla_d^2$  is the Laplace's operator in D dimension[19], which at  $l = 0$  is given by,

$$\nabla_d^2 \equiv \frac{d^2}{dr^2} + \frac{d-1}{r} \frac{d}{dr} \quad (24)$$

Now,  $\langle H \rangle$  can be expressed as (with only Coulomb term in the potential in compact extra dimension),

$$\langle H \rangle = \left\langle -\frac{\nabla_d^2}{2\mu} \right\rangle + \left\langle \frac{-A}{r_d} \right\rangle \quad (25)$$

$$\langle H \rangle = \left\langle -\frac{\nabla_3^2}{2\mu} \right\rangle + \left\langle -\frac{1}{2\mu} \frac{\delta^2}{\delta y^2} \right\rangle + \left\langle -\frac{A}{r_d} \right\rangle = \langle H_1 \rangle + \langle H_2 \rangle + \langle H_3 \rangle \quad (26)$$

Where first two terms are the kinematic contribution and third term corresponds to potential contribution. Now putting correct D-dimensional wave function as given in (20) and using coupling constant from equation (10) in equation (26) we get,

$$\langle H_1 \rangle = \left\langle -\frac{\nabla_3^2}{2\mu} \right\rangle = \frac{9}{32\mu} \left( \frac{\mu\pi}{12} \right)^2 \quad (27)$$

For the only compact extra dimension,  $\psi = N_d e^{-\mu A y}$ , and with it we get,

$$\langle H_2 \rangle = \left\langle -\frac{1}{2\mu} \frac{\delta^2}{\delta y^2} \right\rangle = \frac{N_d^2 A}{4} (1 + 2\mu AL) \quad (28)$$

And the potential term,

$$\langle H_3 \rangle = \left\langle -\frac{A}{r_d} \right\rangle = N_d^2 A d C_d \left[ \frac{1}{2} \frac{\sqrt{\pi}}{\sqrt{\mu A}} \frac{\Gamma(5\sigma + \frac{1}{2})}{(2\mu A)^{(5\sigma + \frac{1}{2})}} + \frac{1}{2} \sqrt{\pi} L \frac{\Gamma(5\sigma)}{(2\mu A)^{5\sigma}} \right] \quad (29)$$

neglecting higher orders of L, since L is very small. Then we get the final result,

$$\langle H \rangle = \langle H_1 \rangle + \langle H_2 \rangle + \langle H_3 \rangle = \frac{9}{32\mu} \left( \frac{\mu\pi}{12} \right)^2 + \frac{N_d^2 A}{4} (1 + 2\mu AL) + N_d^2 A d C_d \left[ \frac{1}{2} \frac{\sqrt{\pi}}{\sqrt{\mu A}} \frac{\Gamma(5\sigma + \frac{1}{2})}{(2\mu A)^{(5\sigma + \frac{1}{2})}} + \frac{1}{2} \sqrt{\pi} L \frac{\Gamma(5\sigma)}{(2\mu A)^{5\sigma}} \right] \quad (30)$$

We can write,

$$\langle H \rangle = F(\mu) + G(\mu L) \quad (31)$$

where,  $F(\mu)$  is L independent with  $F(\mu) = \frac{9}{32\mu}(\frac{\mu\pi}{12})^2$  and

$$G(\mu L) = \frac{N_d^2 A}{4}(1 + 2\mu AL) + N_d^2 A d C_d [\frac{1}{2} \frac{\sqrt{\pi}}{\sqrt{\mu A}} \frac{\Gamma(5\sigma + \frac{1}{2})}{(2\mu A)^{(5\sigma + \frac{1}{2})}} + \frac{1}{2} \sqrt{\pi} L \frac{\Gamma(5\sigma)}{(2\mu A)^{5\sigma}}], \text{ is L dependent.}$$

Since  $G(\mu L)$  is not explicitly a linear function of L, equation(31) shows that variation of mass with size of extra-dimension is not linear.

## 4 Results:

### 4.1 Estimation of QCD Bohr radii of heavy flavoured mesons and comparision with varius theory and experimental models:

In table(2), we show that QCD Bohr radii of various heavy flavoured quark anti-quark systems in 'Fermi(fm)' and then compare with the experimental and theoretical limit of extra-dimension in table(3). As the obtained QCD Bohr radii are always well within the corresponding limits of table(3), it is therefore, tempting to assume that physical mesons too might be stable within such QCD Bohr radii and see its consequences in their masses.

Table 2: QCD Bohr radius

Mesons	Reduced mass(Gev)	$a_0 _{QCD} = \frac{3}{16\mu\alpha_s}(m)$
$B^0(d\bar{b})$	0.1733	$4.9 \times 10^{-15}$
$B_s^0(s\bar{b})$	0.23727	$3.59 \times 10^{-15}$
$B_c^+(c\bar{b})$	1.02254	$0.83 \times 10^{-15}$
$D^-(\bar{c}d)$	0.15826	$3.03 \times 10^{-15}$
$D_s^+(c\bar{s})$	0.2099	$2.29 \times 10^{-15}$

Table 3: Different experimental and theoretical limit on the size of extra dimension

Experiment and Models	Limit on the size of extra-dimension (m)
Fermi-LAT[21]	$8 \times 10^{-9}m$ (LED)
LEP-I[20]	$4.5 \times 10^{-14}m$
ADD [2]	$\sim 10^{-3}m$
Martin Bures[6]	$\leq \frac{a_0}{4}(0.13225 \times 10^{-10})m$
ALEPH,DELPHI,OPAL[20]	$\sim 6 \times 10^{-18}m$
RS[3]	$2 \times 10^{-9}m$



## 4.2 Variation of mass of mesons with size of extra-dimension :

With the expression obtained for  $\langle H \rangle$ , we calculate the mass of heavy flavoured meson [5],[12],  $B(\bar{b}c)$ . We take the value of ' $L$ ' according to the condition  $R_{QCD} \leq \frac{3}{16\mu\alpha_s}$ , [6]. In table(4), we show the variation of mass of  $B(\bar{b}c)$  meson with the size of finite extra-dimension, which is well within the corresponding QCD Bohr radii, 0.83fm (table(2)). This table(4) shows that as size of extra-dimension increases, mass of meson also increases. Further, for  $L=0$ , at  $d=3$ , the calculated mass is above the experimental value suggesting that effectively confinement effect reduces mass of a meson in  $3D$ . We therefore raise the question if such reduction of mass due to the confinement effect can be obtained equivalently through the assumption that gluon effect can as well propagate to extra-dimension within the QCD Bohr radii. So, in table(5), we generalize the effect of confinement through the compact extra dimension of suitable size, for  $B(\bar{b}c)$ . This table(5) shows that mass of  $B(\bar{b}c)$  meson can be obtained with the assumption of extra-dimension of size  $L$  where the gluon can effectively propagate without any confinement effect. Similar analysis is done for  $\tau(\bar{b}b)$  in table(6), at which experimental mass is obtained without confinement effect. The input parameters for numerical calculations used are  $m_b = 4.66\text{Gev}$ ,  $m_c = 1.31\text{Gev}$  and  $\alpha_s$  values 0.39 and 0.22 for c-scale and b-scale respectively.

Table 4: Variation of mass of  $B(\bar{b}c)$  meson with size of extra-dimension without confinement:

$L(fm)$	$m(q) + m(\bar{q})$	$M_P(Gev)$	$Exp.mass(Gev)$
0.01	5.97	6.4343	$6.27 \pm 0.006$
0.02	5.97	6.4474	
0.03	5.97	6.4578	
0.04	5.97	6.469	
0.05	5.97	6.4821	

Table 5: Mass of  $B(\bar{b}c)$  meson without confinement in finite extra-dimension:

$SizeL(fm)$	$m(q) + m(\bar{q})$	$M_P(Gev)$
0.005	5.97	6.24
0.006	5.97	6.247
0.007	5.97	6.25
0.008	5.97	6.26
0.009	5.97	6.27

Table 6: Mass of  $\tau(\bar{b}b)$ meson without confinement in finite extra-dimension:

$Size L(fm)$	$m(q) + m(\bar{q})$	$M_P(Gev)$
0.1	9.32	9.479
0.12	9.32	9.467
0.15	9.32	9.46
0.18	9.32	9.427
0.2	9.32	9.40

## 5 Conclusion and Discussion:

In this paper the recent result that the H-atom can be stable in finite extra dimension if the size of extra dimension does not exceed  $\frac{a_0}{4}$  [6], where,  $a_0$  is the Bohr radius, has been extended to QCD and corresponding QCD Bohr radii for quark-antiquark systems are evaluated. It is found that the QCD Bohr radii of various Heavy Flavoured mesons are well within the present theoretical and experimental limit of size of extra-dimension. We then study its consequences in their masses using an effective string inspired potential model [5]. Within the uncertainty of masses of known Heavy Flavoured mesons the allowed range of extra dimension is  $L \leq 10^{-16}m$ , which is also well below the present theoretical and experimental limit, but far above the Planck length  $\simeq 1.5 \times 10^{-35}$  m. It is interesting to note that the potential suggested by us has got strong resemblance to the corresponding potential in deconfined phase in QGP [22] and indicates plausible relationship between the confinement effect and the coulomb potential in finite extra-dimension. Hence our result does not contradict the present notion that the extra degrees of freedom for gluons and quarks in extra-dimension is not yet discovered.

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